

Landau damping of transverse quadrupole oscillations of an elongated Bose-Einstein condensate

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We study the interaction between low-lying transverse collective oscillations and thermal excitations of an elongated Bose-Einstein condensate by means of perturbation theory. We consider a cylindrically trapped condensate and calculate the transverse elementary excitations at zero temperature by solving the linearized Gross-Pitaevskii equations in two dimensions. We use them to calculate the matrix elements between thermal excited states coupled with the quasi-2D collective modes. The Landau damping of transverse collective modes is investigated as a function of temperature. At low temperatures, the damping rate due to the Landau decay mechanism is in agreement with the experimental data for the decay of the transverse quadrupole mode, but it is too small to explain the slow experimental decay of the transverse breathing mode. The reason for this discrepancy is discussed.

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I. INTRODUCTION

Transverse collective oscillations have been recently excited in the radial plane of an elongated cylindrically symmetric condensate [1]. They are quasi-2D excitations and indeed, at very low temperatures, the measured frequencies of the transverse monopole and quadrupole modes are found to be in very good agreement with the theoretical predictions obtained by linearizing the Gross-Pitaevskii equation for a trapped two-dimensional (2D) system. Moreover, it has been experimentally found that the transverse breathing mode of an elongated condensate exhibits unique features. First, the excitation frequency is close to twice the radial trapping frequency, it is almost independent of the strength of the two-body interaction and the number of particles, and it is nearly independent of temperature. And second, the transverse breathing mode has a very small damping rate compared to other modes. The measured damping rate of the transverse quadrupole mode is approximately one order of magnitude larger than the one of the transverse breathing mode. This is in contrast to the 3D case, where both monopole and quadrupole modes have similar decay rates. These striking properties experimentally found for

the transverse breathing mode are in agreement with the unique features predicted for the breathing mode in 2D isotropic condensates [2–4]: its frequency is a universal quantity and it is an undamped mode. One can conclude that these features are characteristic of the reduced dimensionality thus, providing a demonstration of the two-dimensional nature of the transverse oscillations of an elongated condensate. Therefore, a highly anisotropic cigar-shaped trap can be considered as a first approximation as an infinite cylinder along the z -direction.

In the present paper we consider a cylindrical condensate and calculate at zero temperature the transverse spectrum of excitations, by solving the linearized Gross-Pitaevskii equation in 2D, and the modes with non-zero momentum k along the longitudinal axis, as well. The main purpose is to investigate the coupling between quasi-2D (transverse) collective oscillations and thermal excitations in the collisionless regime. We study the Landau damping as a decay mechanism of transverse collective modes as a function of temperature, in which a transverse collective oscillation is annihilated in a collision with a thermal excitation to give rise to another excitation.

This paper is organized as follows. In Sec. II we calculate the transverse spectrum of excitations of a cylindrical condensate by solving the linearized Gross-Pitaevskii equation in 2D within the Bogoliubov theory. We obtain also the modes with non-zero k . In Sec. III we briefly recall the perturbation theory for the interaction between collective modes of a condensate and thermal excitations (developed in Ref. [5]). We apply this theory to calculate the decay rate due to Landau damping of a transverse oscillation. In Sec. IV we discuss the main results for the transverse quadrupole mode. The situation with the singular transverse breathing mode is also discussed.

II. ELEMENTARY EXCITATIONS OF A CYLINDRICALLY TRAPPED CONDENSATE

At low temperature, the elementary excitations of a trapped weakly interacting degenerate Bose gas are described by the time-dependent Gross-Pitaevskii (GP)

equation for the order parameter:

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \left(-\frac{\hbar^2 \nabla^2}{2M} + V_{\text{ext}}(\mathbf{r}) + g |\Psi(\mathbf{r}, t)|^2 \right) \Psi(\mathbf{r}, t), \quad (1)$$

where $g = 4\pi\hbar^2 a/M$ is the interaction coupling constant, fixed by the s -wave scattering length a , M is the atomic mass and the order parameter is normalized to the number of atoms in the condensate N . The confining potential is usually cylindrically symmetric and it is given by $V_{\text{ext}}(r_\perp, z) = M(\omega_\perp^2 r_\perp^2 + \omega_z^2 z^2)/2$, with $r_\perp^2 = x^2 + y^2$. The ratio between axial (ω_z) and radial (ω_\perp) trapping frequencies defines the anisotropy parameter of the trap $\lambda = \omega_z/\omega_\perp$.

Experimentally, in Ref. [1], the transverse modes have been excited in the radial plane of a highly anisotropic cigar-shaped trap, with $\lambda \sim 0.0646$. Thus, as a reasonable approximation, we consider an idealized cylindrical trap that is uniform in the z -direction ($\omega_z = 0$) and has isotropic trapping potential in the radial plane

$$V_{\text{ext}}(r_\perp) = \frac{1}{2} M \omega_\perp^2 r_\perp^2. \quad (2)$$

The harmonic trap frequency ω_\perp provides a typical length scale for the system, $a_\perp = (\hbar/M\omega_\perp)^{1/2}$. We assume a cylindrical condensate with large longitudinal size L compared to the radial size R , and with a number of atoms per unit length $N_\perp = N/L$.

At low temperatures, the dynamics of the condensate is described by the linearized time-dependent GP equation. Assuming small oscillations of the order parameter around the ground-state value

$$\Psi(\mathbf{r}, t) = e^{-i\mu t/\hbar} [\Psi_0(r_\perp) + \delta\Psi(\mathbf{r}, t)], \quad (3)$$

where μ is the chemical potential, the normal modes of the condensate can be found by seeking fluctuations such as

$$\delta\Psi(\mathbf{r}, t) = u(\mathbf{r})e^{-i\omega t} + v^*(\mathbf{r})e^{i\omega t}. \quad (4)$$

The functions u and v are the ‘‘particle’’ and ‘‘hole’’ components characterizing the Bogoliubov transformations, and ω is the frequency of the corresponding excited state.

The transverse excitations of a highly elongated condensate can be obtained by neglecting oscillations along the longitudinal direction and thus, looking for fluctuations in the transverse plane of the form [6,7]

$$\delta\Psi(r_\perp, \theta, t) = U(r_\perp) e^{im\theta} e^{-i\omega t} + V^*(r_\perp) e^{-im\theta} e^{i\omega t}, \quad (5)$$

where we have used cylindrical coordinates (r_\perp, θ, z) .

Transverse excitations are quasi-2D modes. To calculate Landau damping we must also consider thermal excitations having a quasi-continuous spectrum at $L \gg R$ with the finite longitudinal wave vector k along z direction. To study these modes we look for solutions that correspond to propagating waves of the form [6,8]

$$\delta\Psi(\mathbf{r}, t) = U_k(r_\perp) e^{i(m\theta + kz)} e^{-i\omega t} + V_k^*(r_\perp) e^{-i(m\theta + kz)} e^{i\omega t}. \quad (6)$$

Inserting (3), (4) and (6) in Eq. (1) and retaining terms up to first order in u and v , it follows three equations. The first one is the nonlinear equation for the order parameter of the ground state,

$$[H_0 + g\Psi_0^2(r_\perp)] \Psi_0(r_\perp) = \mu \Psi_0(r_\perp), \quad (7)$$

where $H_0 = -(\hbar^2/2M)\nabla_{r_\perp}^2 + V_{\text{ext}}(r_\perp)$, and $\nabla_{r_\perp}^2 = d^2/dr_\perp^2 + r_\perp^{-1}d/dr_\perp$; while $U_k(r_\perp)$ and $V_k(r_\perp)$ obey the following coupled equations [9]:

$$\hbar\omega U_k(r_\perp) = \left[H_0 + \frac{\hbar^2}{2M} \left(\frac{m^2}{r_\perp^2} + k^2 \right) - \mu + 2g\Psi_0^2 \right] U_k(r_\perp) + g\Psi_0^2 V_k(r_\perp) \quad (8)$$

$$-\hbar\omega V_k(r_\perp) = \left[H_0 + \frac{\hbar^2}{2M} \left(\frac{m^2}{r_\perp^2} + k^2 \right) - \mu + 2g\Psi_0^2 \right] V_k(r_\perp) + g\Psi_0^2 U_k(r_\perp). \quad (9)$$

We solve the linearized GP equations Eqs. (7-9) at zero temperature, to obtain the ground state wave function Ψ_0 , the eigenfrequencies $\omega(k)$ and hence the energies $E = \hbar\omega$ of the excitations, as well as the corresponding functions (u, v) .

In a cylindrical trap the modes of the condensate are labeled by $i = (n, m, k)$, where n is the number of nodes in the radial solution, m is the z -projection of the orbital angular momentum, and k is the longitudinal wave vector. The quantum numbers n and m are discrete ($n = 0, 1, 2, \dots$, and $m = 0, \pm 1, \pm 2, \dots$) whereas k for highly elongated condensates ($L \gg R$) is a real number, and thus, it is a continuous index. Moreover, it can be seen from Eqs. (8) and (9), that levels with $m = \pm |m|$ or $k = \pm |k|$ are exactly degenerated.

The excitation spectrum of a cylindrical condensate presents different branches labeled by (m, n) that are continuous and increasing functions of k [8]. For a given mode, i.e., fixed n and m , the frequency of excitation is $\omega(k)$.

We have numerically checked from Eqs. (8) and (9) that the state with $(m = 0, n = 1)$ and $k = 0$ corresponds to the transverse breathing mode with excitation frequency equal to $2\omega_\perp$, and that it is independent of the strength of the two-body interaction potential and the number of particles. Of course, according to the universal nature of this mode noted above, this general result is also valid in the Thomas-Fermi (TF) limit, where the dispersion relation can be found analytically [6,7]. However, the functions u and v for this mode do not coincide in general with their Thomas-Fermi limit.

The state with $(m = 2, n = 0)$ and $k = 0$ is a transverse quadrupole mode whose frequency depends on the two-body interaction, ranging from the TF value $\sqrt{2}\omega_\perp$, valid for large condensates ($aN_\perp \gg 1$ or equivalently $\mu \gg \hbar\omega_\perp$), to the noninteracting value $2\omega_\perp$. Therefore,

the transverse quadrupole has to be obtained numerically by solving Eqs. (8) and (9) in each particular value of the dimensionless parameter aN_\perp . In the present work, we use the numerical solutions to calculate the Landau damping of collective modes.

It is well known that the dipole mode ($m = 1, n = 0$) is unaffected by two-body interactions due to the translational invariance of the interatomic force which cannot affect the motion of the center of mass [9] and its excitation frequency is $1\omega_\perp$. It is worth stressing that the independence of the interaction of the transverse monopole mode, is a unique property of 2D systems, related to the presence of a hidden symmetry of the problem described by the two-dimensional Lorentz group $SO(2,1)$ [4].

III. LANDAU DAMPING

Let us consider a collective mode with frequency Ω_{osc} , and the excited states i, j available by thermal activation, with energies E_i and E_j , respectively. Suppose that this collective mode has been excited and, therefore, the condensate oscillates with the corresponding frequency Ω_{osc} . Due to interaction effects, the thermal cloud of excitations can either absorb or emit quanta of this mode producing a damping of the collective oscillation. We want to study the decay process in which a quantum of oscillation $\hbar\Omega_{\text{osc}}$ is annihilated (created) and the i -th excitation is transformed into the j -th one (or viceversa). The energy is conserved during the transition process, therefore

$$E_j = E_i + \hbar\Omega_{\text{osc}}, \quad (10)$$

where we assume that $E_j > E_i$. This mechanism is known as Landau damping. Another possible decay mechanism, also due to the coupling between collective and thermal excitations, is the Beliaev damping [10], which is based on the decay of an elementary excitation into a pair of excitations.

Let us define the dissipation rate γ through the following relation between the energy of the system E and its dissipation \dot{E} :

$$\dot{E} = -2\gamma E. \quad (11)$$

Assuming that the damping is small, one can use perturbation theory to calculate the probabilities for the transition between the i -th excitation and the j -th one, both available by thermal activation, yielding the following expression for the Landau damping rate [5,11]:

$$\frac{\gamma}{\Omega_{\text{osc}}} = \sum_{ij} \gamma_{ij} \delta(\omega_{ij} - \Omega_{\text{osc}}), \quad (12)$$

where the transition frequencies $\omega_{ij} = (E_j - E_i)/\hbar$ are positive, and the delta function ensures the energy conservation during the transition process. We have assumed that the thermal cloud is at thermodynamic equilibrium

and the states i, j are thermally occupied with the usual Bose factor $f_i = [\exp(E_i/k_B T) - 1]^{-1}$. The “damping strength” has the dimensions of a frequency and is given by

$$\gamma_{ij} = \frac{\pi}{\hbar^2 \Omega_{\text{osc}}} |A_{ij}|^2 (f_i - f_j). \quad (13)$$

The matrix element that couples the low-energy collective mode ($u_{\text{osc}}, v_{\text{osc}}$) with the higher energy single-particle excitations (for which we use the indices i, j) is [5]

$$A_{ij} = 2g \int d\mathbf{r} \Psi_0 [(u_j^* v_i + v_j^* v_i + u_j^* u_i) u_{\text{osc}} + (v_j^* u_i + v_j^* v_i + u_j^* u_i) v_{\text{osc}}]. \quad (14)$$

In this work we calculate the quantities γ_{ij} by using the numerical solutions u and v of Eqs. (7-9) into the integrals (14), avoiding the use of further approximations in the spectrum of excitations. In cylindrically symmetric traps, $i = (n, m, k)$ and from (6) the longitudinal excitations are $u_{n m k}(\mathbf{r}) = U_{n m k}(r_\perp) e^{im\theta} e^{ikz}$ and the energies $E_{n m k}$ are not degenerate.

We are interested in the decay of transverse collective excitations (5) $u_{\text{osc}}(r_\perp, \theta) = U_{\text{osc}}(r_\perp) e^{im_{\text{osc}}\theta}$, due to the coupling with thermally excited states (i, j) . The functions u_{osc} and v_{osc} do not depend on z , and from Eq. (14) it is straightforward to see that the matrix element A_{ij} couples only energy levels (i, j) with the same quantum number $k \equiv k_i = k_j$, and $\Delta m \equiv |m_i - m_j| = m_{\text{osc}}$. To calculate the damping rate γ of the collective oscillation we have to sum up over all couples of excited states (i, j) , or equivalently, over all pairs of sets $(n_i m_i k, n_j m_j k)$, that satisfy the above conditions. Since k is a continuous quantum number, the sum over k becomes an integral and Eq. (12) yields

$$\frac{\gamma}{\Omega_{\text{osc}}} = \sum_{n_i n_j m_i m_j} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \gamma_{ij} \delta(\omega_{ij} - \Omega_{\text{osc}}), \quad (15)$$

where the sum is restricted to pairs that satisfy $\Delta m = m_{\text{osc}}$. The frequency of a mode is a continuous function of k , therefore the transition frequency is $\omega_{ij} = \omega_{ij}(k)$. Then, in Eq. (15) the integration over k yields

$$\frac{\gamma}{\Omega_{\text{osc}}} = \frac{1}{2\pi} \sum_{\tilde{k}} \sum_{n_i n_j m_i m_j} \gamma_{ij} \left[\left| \frac{\partial \omega_i}{\partial k} - \frac{\partial \omega_j}{\partial k} \right|_{\tilde{k}} \right]^{-1} \quad (16)$$

where \tilde{k} is the wave vector in which the conservation of energy is verified, i.e., $|\omega_i(\tilde{k}) - \omega_j(\tilde{k})| = \Omega_{\text{osc}}$, and we have used the known property of delta functions

$$\int_{-\infty}^{\infty} dk \delta(g(k)) = \sum_{\tilde{k}} \frac{\delta(k - \tilde{k})}{|g'(\tilde{k})|}. \quad (17)$$

IV. RESULTS

We want to calculate the characteristic decay rates due to Landau damping of the transverse low-lying collective modes of an elongated Bose-Einstein condensate as a function of T . We consider the collective excitations in the collisionless regime, which is achieved at low enough temperatures.

For a fixed number of trapped atoms, the number of atoms in the condensate depends on temperature. At zero temperature the quantum depletion is negligible and all the atoms can be assumed to be in the condensate [12]. At finite temperature the condensate atoms coexist with the thermal bath. However, at low enough temperatures the excitation spectrum can be safely calculated by neglecting the coupling between the condensate and thermal atoms [9]. It means that the excitation energies at a given T can be obtained within Bogoliubov theory at $T = 0$ normalizing the number of condensate atoms to the corresponding condensate fraction at that temperature and assuming that they are thermally occupied with the Bose factor.

In order to present numerical results we choose a gas of ^{87}Rb atoms (scattering length $a = 5.82 \times 10^{-7}$ cm) confined in an elongated cylindrical trap with frequencies $\omega_z \approx 0$ and $\omega_\perp = 219 \times 2\pi$ Hz, that corresponds to an oscillator length $a_\perp = 0.729 \times 10^{-4}$ cm. The number of condensate atoms per unit length is taken to be $N_\perp = N/L = 2800 a_\perp^{-1}$. These conditions are close to the experimental ones of Ref. [1].

We have solved the linearized GP equations to obtain an exact description of the ground state $\Psi_0(r_\perp)$ and the normal modes of the condensate within Bogoliubov theory without using the Thomas-Fermi or Hartree-Fock approximations. We have obtained the following numerical results: $\mu = 9.526 \hbar\omega_\perp$, and the excitation frequencies of the transverse monopole $\Omega_M = 2.0\omega_\perp$, and of the transverse quadrupole mode $\Omega_Q = 1.436 \hbar\omega_\perp$. We have calculated the branches of the excitation spectrum of the cylindrical condensate, labeled by (m, n) as a function of k .

To calculate the damping rate of a collective mode Ω_{osc} we have to obtain, first, all pairs of levels (i, j) of the excitation spectrum that satisfy both the energy conservation of the transition process $E_j - E_i = \hbar\Omega_{\text{osc}}$, and the selection rules given by the particular collective mode under study. And then, calculate the corresponding matrix elements. We have restricted our calculations to levels with energy $E \leq 7\mu$. The contribution of higher excited levels can be neglected since their occupation becomes negligible in the range of temperatures we have considered.

It is worth recalling that the dipole mode is undamped [9] since it is not affected by two-body interactions. We have checked that within our formalism the Landau damping of the transverse dipole mode is zero, as expected.

First of all, we have calculated the Landau damping

of the transverse quadrupole mode ($k = 0$, $m = 2$ and $n = 0$) as a function of T , $\gamma_Q(T)$. For such a mode, the functions u_{osc} and v_{osc} depend on the radial coordinate r_\perp and have also angular dependence ($\sim e^{\pm i2\theta}$). Therefore, from Eq. (14) it follows the selection rules corresponding to transverse quadrupole-like transitions: $\Delta m = |m_j - m_i| = 2$ and $\Delta k = 0$. Only the energy levels (i, j) that satisfy these conditions can be coupled by the interaction. The integration of the radial part has to be done numerically.

Fixed the number of condensate atoms, we have calculated the Landau damping for the transverse quadrupole mode as a function of temperature. At zero temperature we have obtained the excitation spectrum within Bogoliubov theory. We have found that there are 34 pairs of levels that verify the transverse quadrupole selection rules and the energy condition (10) at finite wave vectors, i.e., at $\tilde{k} \neq 0$. We have calculated the matrix element A_{ij} for each allowed transition, and then, the corresponding damping strength γ_{ij} at a given T . Summing up the damping strengths for all transitions, we have obtained the damping rate for the transverse quadrupole oscillation, due to the coupling with thermal excitations.

In Figure 1 we plot the damping rate versus $k_B T / \mu$ for the transverse quadrupole mode for a fixed number of condensate atoms per unit length in a cylindrical trap. As expected, Landau damping increases with temperature since the number of excitations available at thermal equilibrium is larger when T increases. One can distinguish two different regimes, one at very low T ($k_B T \ll \mu$) and the other at higher T , where damping is linear with temperature. However, this linear behaviour appears at relatively small temperature ($k_B T \sim \mu$) in comparison to the homogeneous system [5] where this regime occurs at $k_B T \gg \mu$. For the highly elongated condensate of Ref. [1], it has been measured at $k_B T \simeq 0.7\mu$ a quality factor [13] for the transverse quadrupole mode $Q = \Omega_Q / (2\gamma_Q^{\text{exp}}) \sim 200$ which gives a damping rate $\gamma_Q^{\text{exp}} \sim 10^{-2}\omega_\perp$. This value is in agreement with our calculations of the Landau damping for this mode (see Fig. 1). A more quantitative comparison is not possible at present due to the absence of detailed measurements of the damping of this mode. However, it is reasonable to conclude that the transverse quadrupole mode in elongated condensates decays via Landau damping. It is worth noting that the order of magnitude of the damping rate of the transverse quadrupole mode is the same as the one previously estimated for a uniform gas [5, 14–16], for spherical traps [11] and for anisotropic traps [17, 18] as well.

The Landau damping for other modes can be calculated analogously. However, the physical situation for the decay of the singular transverse monopole mode turns out to be much more complicated. Formally the calculations can be committed in the same way. The quantum numbers for this mode are $k = 0$, $m = 0$ and $n = 1$. The functions u_{osc} and v_{osc} depend only on the radial coordi-

nate r_{\perp} , and the matrix element A_{ij} couples only energy levels (i, j) with the same quantum numbers m and k . That is, the selection rules corresponding to transverse monopole-like transitions are $\Delta k = 0$ (as we have already discussed in the previous section) and $\Delta m = 0$.

From the spectrum of elementary excitations, we have found that there are only 16 pairs of levels that verify the transverse monopole selection rules and the energy condition (10) at $\tilde{k} \neq 0$. The damping strengths associated to the allowed transitions are smaller than the γ_{ij} values associated to quadrupole-like transitions. As a result from our calculations the damping occurs quite small. At $T \simeq 1.5\mu$ we obtain a Landau damping $\gamma_M \simeq 7 \times 10^{-5}\omega_{\perp}$ that is one order of magnitude smaller than the experimental decay [13] $\gamma_M^{\text{exp}} \simeq 6 \times 10^{-4}\omega_{\perp}$ measured in [1]. Thus, Landau damping cannot explain the experimentally observed decay of the transverse breathing mode.

Actually, there are reasons to believe that even this small value of Landau damping obtained in the present calculations, could be larger than the true one. The point is that in our calculations, based on perturbation theory, the thermal cloud is assumed to be in the state of thermodynamic equilibrium and thus one can neglect its motion. This is correct for the case of quadrupole oscillations, where the frequencies of the collective motion of the condensate and the cloud are different. However, due to the unique peculiar nature of the 2D breathing mode, the breathing oscillation of the thermal cloud has the same frequency $2\omega_{\perp}$ as the condensate. Then, if the oscillation is excited by the deformation of the trap, as it takes place in the present experiment [1], the cloud will oscillate in phase with the condensate what will further decrease the Landau damping [19]. Therefore, this quasi-2D monopole mode in highly elongated condensates may decay via another damping mechanism, different from the Landau damping.

After finishing this work it has appeared a preprint by Jackson and Zaremba [21] in which the effect of the collisions between elementary excitations has been taken also into account. This effect is usually small in comparison with the Landau damping. Nevertheless, for this peculiar mode the collisions seems to be the dominant damping mechanism, yielding a decay rate that it is in agreement with the experimentally measured value [1]. Note, however, that the experiments are produced at quite high amplitude of oscillations and non-linear effects can play an important role. For example, as it was noted in [22], the transverse breathing mode exhibits a "parametric" instability due to the decay into two or more excitations with non-zero momentum along the axis. In Ref. [23] it has been considered the process in which the transverse breathing oscillation decays into two excitations propagating along z with momentum k and $-k$. This damping mechanism is active also at zero temperature. But in this case the damping rate depends on the amplitude of the breathing oscillation.

V. SUMMARY

We have investigated the decay of low-lying transverse oscillations of a large cylindrical condensate. First of all, we have calculated the transverse normal modes, as well as the excitations with finite k , of the condensate by solving the linearized time-dependent Gross-Pitaevskii equation. And then, within the formalism of Ref. [11], we have calculated numerically the matrix elements associated with the transitions between excited states allowed by the selection rules of the transverse collective modes. Within a first-order perturbation theory and assuming the thermal cloud to be in thermodynamic equilibrium, we have studied the Landau damping of transverse collective modes due to the coupling with thermal excited levels as a function of temperature. For the damping rate of the transverse quadrupole mode, we have found a good agreement with the experimentally measured value of Ref. [1]. Whereas our result for the transverse breathing mode is one order of magnitude smaller than the experimental decay. One can conclude that in a highly elongated cylindrically symmetric condensate, transverse quadrupole oscillations decay via Landau damping mechanism, but the transverse breathing mode which has an anomalously small measured damping rate, may decay via other damping mechanisms.

VI. ACKNOWLEDGMENTS

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- [19] Note that the "perturbative" calculations of the Landau damping can correspond to a different type of experiment, in which the oscillations of the condensate are excited by the action of a thin laser beam on the condensate, but not on the thermal cloud. In this case the "perturbative" calculations will describe the initial stage of the process, when the cloud is still at rest. As it was presented in the lecture by E. Zaremba [20], the Landau damping corresponding to this type of experiment was calculated using the method developed in Ref. [21] in an elongated condensate with the same experimental parameters of Ref. [1]. However, they have obtained larger values for the Landau damping of the transverse breathing mode in comparison with our results. This discrepancy could be related to the semiclassical nature of their method [21], which does not take into account the discrete nature of the energy spectrum, and also to the fact that they consider a cigar-shaped condensate instead of an infinite cylinder as in the present calculations. The finite value of the longitudinal frequency ω_z could be important for this transverse breathing mode. Therefore, this is still an interesting open problem.
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FIG. 1. Dimensionless damping rate of the transverse quadrupole mode γ_Q/ω_\perp as a function of $k_B T/\mu$ for a cylindrical condensate with $N_\perp = 2800 a_\perp^{-1}$ atoms of ^{87}Rb per unit length.

